HW: 5.6/1-35 (the ones you didn't do already)

5.6/15)

$N = 5000\left(25 + te^{-t/20}\right)$

a) $0 \le t \le 100$

$N' = 5000\left(e^{-t/20} + t\left(-\frac{1}{20}e^{-t/20}\right)\right)$

$N' = 5000e^{-t/20}\left(1 - \frac{1}{20}t\right) = 0$

$1 = \frac{1}{20}t$ or $t = 20$ [critical #]

b) end behavior or endpoints

$N(0) = 5000(25 + 0(\text{who cares}))$ $N = 5000\left(25 + te^{-t/20}\right)$

$= 5000(25)$

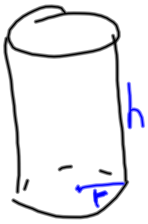
$\cdot \frac{20}{e}$ ☆ $N(20) = 5000(25 + 20e^{-1}) = 5000 \cdot 25 + 5000 \cdot \frac{20}{e}$

$\cdot \frac{20}{e} \cdot \frac{5}{e^4}$ $N(100) = 5000(25 + 100e^{-5}) = 5000 \cdot 25 + 5000\left(\frac{100}{e^5}\right)$

ABS MIN (occurs at $t=0$) is 125,000

ABS MAX (occurs at $t=20$) is $125000 + \frac{100000}{e}$

5.6/27) A closed cylindrical CAN is to have a volume of V. when does this happen?



$V = \pi r^2 \cdot h$ $SA = 2\pi r^2 + 2\pi r h$

$\frac{V}{\pi r^2} = h$ (var) (var)

minimize surface area

$\frac{V}{\pi r^2} = h$

$r = \sqrt{\frac{V}{\pi h}}$

$SA = 2\pi r(r + h)$

$SA = 2\pi r\left(r + \frac{V}{\pi r^2}\right) = 2\pi r^2 + \frac{2V}{r}$

$SA' = 4\pi r - \frac{2V}{r^2}$

① critical #s

SA' is undefined $(r = 0)$

$SA' = 0$

$4\pi r - \frac{2V}{r^2} = 0$

$4\pi r^3 = 2V$

$r^3 = \frac{V}{2\pi}$; so $r = \sqrt[3]{\frac{V}{2\pi}}$

find r when SA is min

and $\sqrt{\frac{V}{\pi h}} = \sqrt[3]{\frac{V}{2\pi}}$

$\frac{2^{1/3}\pi^{1/3} V^{1/2}}{\pi^{1/2} V^{1/3}} = \sqrt{h}$

$\frac{2^{2/3} V^{1/3}}{\pi^{1/3}} = \frac{2^{2/3}\pi^{2/3} V}{\pi V^{2/3}} = h$

find h when SA is minimum

#3 find ratio $\frac{h}{r}$

$\frac{h}{r} = \frac{\frac{2^{2/3} V^{1/3}}{\pi^{1/3}}}{\frac{V^{\frac{1}{3}}}{2^{\frac{1}{3}}\pi^{1/3}}}$

$= 2^{\frac{2}{3}} \cdot 2^{\frac{1}{3}} = 2$

$\frac{h}{r} = 2$

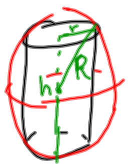
Calling 911 is fun

\- Bin

$$h = \frac{2^{\frac{2}{3}} V^{\frac{1}{3}}}{\pi^{1/3}} \qquad d = 2r = 2\,\frac{V^{\frac{1}{3}}}{2^{\frac{1}{3}}\pi^{\frac{1}{3}}} = \frac{2^{2/3} V^{\frac{1}{3}}}{\pi^{\frac{1}{3}}}$$

same

25 (5.6)



We know:
$2h$ is the height of the con
r is the radius of the cyl-in-the-sphere
R^* is the radius of the sphere

Goal:
maximize surface area.

$$SA = 2\pi r^2 + 4\pi r h$$
$$= 2\pi r(r+2h)$$
$$= 2\pi r\left(r + 2\sqrt{R^2-r^2}\right)$$

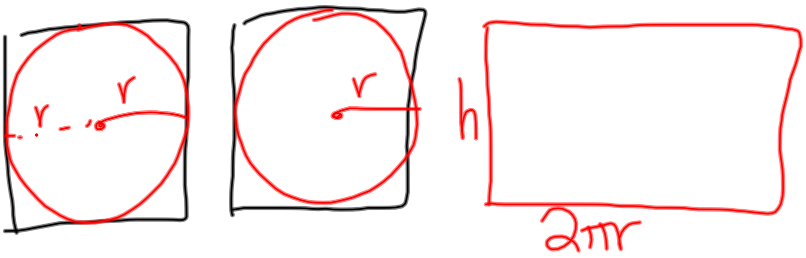
$$R^2 = h^2 + r^2 \Rightarrow h^2 = R^2 - r^2$$
$$h = \sqrt{R^2-r^2}$$

$$SA' = 2\pi\left(r+2\sqrt{R^2-r^2}\right) + 2\pi r\left(1 + (R^2-r^2)^{-\frac{1}{2}}(-2r)\right)$$

$$\frac{dSA}{dr} = 2\pi r + 4\pi\sqrt{R^2-r^2} + 2\pi r - \frac{4\pi r^2}{\sqrt{R^2-r^2}}$$
$$= 4\pi r + 4\pi\sqrt{R^2-r^2} - \frac{4\pi r^2}{\sqrt{R^2-r^2}}$$

$SA' = 0 \Rightarrow$
$$4\pi r + 4\pi\sqrt{R^2-r^2} - \frac{4\pi r^2}{\sqrt{R^2-r^2}} = 0$$
$$4\pi\left(r + \sqrt{R^2-r^2} - \frac{r^2}{\sqrt{R^2-r^2}}\right) = 0$$
so
$$r + \sqrt{R^2-r^2} - \frac{r^2}{\sqrt{R^2-r^2}} = 0$$
$$r\sqrt{R^2-r^2} + (R^2-r^2) - r^2 = 0$$
$$\sqrt{R^2-r^2} = \frac{2r^2-R^2}{r} \Rightarrow R^2 - r^2 = \frac{4r^4 - 4r^2R^2 + R^4}{r^2}$$
$$R^2 = \frac{4r^4}{r^2} - 4R^2 + \frac{R^4}{r^2} + r^2 = 5r^2 - 4R^2 + \frac{R^4}{r^2}$$
$$+4R^2 \qquad\qquad +4R^2$$
$$5R^2 = 5r^2 + \frac{R^4}{r^2}$$
$$5R^2r^2 = 5r^4 + R^4 \Rightarrow \boxed{0 = 5r^4 - 5R^2r^2 + R^4}$$
$$r^2 = \frac{5R^2 \pm \sqrt{25R^4 - 4(5)(R^4)}}{10}$$
$$r^2 = \frac{5R^2 \pm \sqrt{5}R^2}{10}.$$

5.6 / 30)



we know $V^* = \pi r^2 h$

$\boxed{\text{minimize material used}}$

$M = 2\pi r h + 2(2r)^2 = 2\pi r h + 4r^2$

but $h = \frac{V}{\pi r^2}$

so

$M = 2\pi r\left(\frac{V}{\pi r^2}\right) + 4r^2 = \frac{2V}{r} + 4r^2$

$M' = -\frac{2V}{r^2} + 8r = 0$

$-2V + 8r^3 = 0$ so $8r^3 = 2V$

$r^3 = \frac{V}{4}$

$r = \left(\frac{V}{4}\right)^{1/3}$

Osprey